Topologically robust transport of entangled photons in a 2D photonic system

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We theoretically study transport of time-bin entangled photon pairs in a two-dimensional topological photonic system of coupled ring resonators. This system implements the integer quantum Hall model using a synthetic gauge field and exhibits topologically robust edge states. We show that the topological edge states provide a robust channel for on-chip quantum communication when the information is encoded in temporal correlations of photons. In contrast to edge states, transport through bulk states does not preserve these correlations and can lead to significant unwanted bunching or anti-bunching of photons. We also compare the transport of entangled two-photon states to separable two-photon states and show that the entangled states are more fragile. Furthermore, we study the effect of disorder on the quantum transport properties; while the edge transport remains robust, bulk transport is very susceptible and, in the limit of strong disorder, bulk states become localized. We show that this localization is manifested as an enhanced bunching/anti-bunching of photons.

I. INTRODUCTION

Recently, there has been a significant surge of interest in investigating topological photonic systems implemented using synthetic gauge fields \cite{1,2}. These systems exhibit topologically robust edge states which are protected against backscattering caused by disorder. Topological edge states have been observed in photonic systems, using coupled ring resonators \cite{3,4} and helical waveguides \cite{5} in the optical domain, and also using metamaterials \cite{6,10} in the microwave domain. Many other interesting platforms have been proposed to realize photonic edge states \cite{11,15}. These states are characterized by a topologically invariant integer, the winding number, which has been measured recently in photonic systems \cite{16,19}. Moreover, the topological robustness of photonic edge states against disorder has been quantitatively established, which merits their use as robust on-chip communication channels and delay lines \cite{6,20}. However, investigations of topological robustness in photonic systems have so far relied on observing classical transport parameters such as transmission and delay statistics. Here, we study quantum transport properties of non-classical light in a two-dimensional (2D) topological photonic system. In contrast to previous works where the input was a classical state, here, the input is a quantum state of two time-bin entangled photons \cite{21,23}.

Our topological system is a 2D lattice of coupled ring resonators, shown in Fig. 1(a). A uniform synthetic magnetic field is introduced in this system by appropriately positioning the ring resonators (Fig. 1(b)) \cite{3,4}. This system simulates the integer quantum Hall model and its transmission spectrum is divided into bulk bands separated by topologically non-trivial edge bands where the corresponding states propagate around the edge of the system \cite{3,4}. We show that the transport through edge band preserves temporal correlations of the input photons whereas bulk transport significantly distorts these correlations, even in the absence of disorder. In particular, for bulk transport, temporally anti-bunched photons at the input can bunch at the output and vice-versa. We compare these results with the case of a separable two-photon state and show that this bunching/antibunching is more prominent for the entangled two-photon state, indicating that the correlated quantum states is more fragile. Furthermore, we study the effect of disorder on the quantum transport. Using the $\Psi^+$ Bell state as an example, where the two photons arrive at the input at different times, we show that disorder leads to temporal bunching of photons at the output and this bunching increases with disorder strength.

We begin with a brief description of our system of coupled ring resonators and the two-photon input state. Subsequently, we analyze edge and bulk transport of maximally entangled Bell states in a system without disorder and contrast this to transport of separable states. Finally, we study the effect of disorder on the quantum transport.

II. ENTANGLED PHOTONS IN A QUANTUM HALL MODEL

The 2D lattice of evanescently coupled ring resonators is shown in Fig. 1(a) \cite{3,4}. The site resonators (shown in black) are coupled using link resonators (in grey). To introduce a synthetic magnetic field, the link resonators connecting site resonators are vertically shifted (Fig. 1(b)). This vertical shift leads to a direction dependent hopping phase, i.e., a photon hopping to the right travels a longer path in the link resonator and hence accumulates an extra phase $\phi$, compared to a photon hopping to the left. This lattice implements the quantum spin-Hall model, where the two pseudo-spins correspond to clockwise and counterclockwise circulation of photons in the site resonators. However, by appropriately choosing the input port (Fig. 1(a)), we can selectively excite a single pseudo-spin component and simulate the integer quantum Hall model with the tight-binding
Hamiltonian given as

$$H = -J \left( \hat{a}_{x+1,y}^\dagger \hat{a}_{x,y} e^{-i\phi} + \hat{a}_{x,y}^\dagger \hat{a}_{x+1,y} e^{+i\phi} + \hat{a}_{x,y+1}^\dagger \hat{a}_{x,y} + \hat{a}_{x,y}^\dagger \hat{a}_{x+1,y+1} \right),$$

where $J$ is the coupling rate between neighboring lattice sites and $\phi$ is the synthetic magnetic flux threading a single plaquette. $\hat{a}_{x,y}^\dagger$ and $\hat{a}_{x,y}$ are the photon creation and annihilation operators, respectively, at the lattice site $(x, y)$. Fig. 1(c) shows the simulated single-photon transmission spectrum for a $8 \times 8$ lattice, with a magnetic flux $\phi = \frac{2\pi}{N}$ per plaquette. The transmission spectrum is divided into bulk band separated by edge bands \[24\]. The edge bands (shaded in green and red) are associated with topologically non-trivial edge states circulating clockwise (CW) and counterclockwise (CCW) along the system boundary. On the other hand, states in the bulk band (shaded in blue) occupy the bulk of the lattice \[3, 4\].

At the input of this lattice, we couple a time-bin entangled two-photon state of the form

$$|\psi\rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dt_1 dt_2 \psi(t_1, t_2; t_e, t_i) \hat{a}^\dagger(t_1) \hat{a}^\dagger(t_2) |0\rangle,$$

where $(t_e)$ and $(t_i)$ correspond to the early and late time bins in which the photons could arrive and $\hat{a}^\dagger(t)$ is the photon creation operator at time $t$. $\psi(t_1, t_2; t_e, t_i)$ is the two-photon temporal wavefunction and is symmetric under exchange of photons. Note that both the photons are centered around the same carrier frequency. Here, we consider the maximally entangled states - the Bell states. For example, the $\Psi^+$ state is written as

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}} (|\epsilon\rangle_1 |l\rangle_2 + |l\rangle_1 |\epsilon\rangle_2),$$

where $|\epsilon\rangle_{1,2}$ and $|l\rangle_{1,2}$ represent the single-photon states in early and late time bins, respectively. It corresponds to a situation when one photon arrives in the early time-bin $(t_e)$ and the other in the late bin $(t_i)$. Similarly, the other two Bell states

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}} (|\epsilon\rangle_1 |\epsilon\rangle_2 + |l\rangle_1 |l\rangle_2)$$

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}} (|\epsilon\rangle_1 |\epsilon\rangle_2 - |l\rangle_1 |l\rangle_2)$$
are the symmetric and antisymmetric combinations of the two scenarios when both the photons arrive early or both arrive late.

The fourth Bell state $\Psi^−$ is not considered here because it is antisymmetric under exchange of photons. These time-bin entangled two-photon states can be realized in various systems, for example, using spontaneous parametric down conversion or quantum dots [21–23].

Assuming the input single-photon temporal wavefunctions are Gaussian, the two-photon wavefunction for $\Psi^+$ state is given by

$$\Psi^+(t_1, t_2; t_e, t_i) = A \left[ \exp \left( -\frac{(t_1 - t_e)^2}{2\sigma^2} \right) \exp \left( -\frac{(t_2 - t_i)^2}{2\sigma^2} \right) + \exp \left( -\frac{(t_1 - t_i)^2}{2\sigma^2} \right) \exp \left( -\frac{(t_2 - t_e)^2}{2\sigma^2} \right) \right],$$

(6)

where $\sigma$ is the single-photon temporal pulsewidth and $A$ is the normalization factor. Similarly, we can write the wavefunctions for the $\Phi^\pm$ states. The temporal two-photon correlation function $\Gamma(t_1, t_2) = |\psi(t_1, t_2)|^2$ is defined as the probability of finding one photon at time $t_1$ and the other photon at time $t_2$. The correlation functions for the $\Psi^+$ and $\Phi^+$ states at the input are shown in Fig. 2(a,d), where we have normalized the maximum to unity. Throughout this paper, we use $\sigma = 10T_0$, where the inverse of the coupling rate $T_0 = 1/J$ is the relevant time scale in the system. This choice of pulsewidth guarantees that the single-photon spectral width $(J/10)$ is much less than the edge/bulk band widths so that we can study the physics of each band separately. We also select the delay between the two time bins $\tau = t_i - t_e = 30T_0$, such that they have negligible overlap at the input. The maximum value of $\tau$ is set by system size, i.e., $\tau < N_xN_yT_0$, where $N_{x,y}$ is the number of site resonators in the $x, y$ direction. In our case, $N_x = N_y = 8$. 

FIG. 2: (a) Time-correlation ($\Gamma(t_1, t_2)$) for $\Psi^+$ input state, with input pulse width 10 $T_0$, and delay $\tau = 30 T_0$, where $T_0 = 1/J$. (b,c) Simulated correlation function at the output port of a 8 × 8 lattice for CCW and CW edge states, respectively. The delay incurred in the edge states shifts the correlation function diagonally but correlation of the input state is preserved. (d-f) Results for the input state $\Phi^+$. Insets show the transmission spectrum and the path followed by edge states. $\Gamma(t_1, t_2)$ is normalized such that the maximum is unity.
III. RESULTS ON TRANSPORT PROPERTIES OF ENTANLED PHOTONS

Now, we analyze transport of these maximally entangled two photons in a system without disorder. For a linear system with no interactions, the two-photon temporal wavefunction can be calculated using the single-photon temporal wavefunctions at the output (see Appendix). Fig. 3(a-c) shows the simulated temporal correlation function at the input and the output for $\Psi^+$ state as the input, with central carrier frequency in the CCW and CW edge bands ($\omega - \omega_0 = \pm 1.5 J$). We see that for both the CCW and CW edge states, the correlation function is translated along the diagonal by the delay incurred in these paths which is equal to $5T_0$ and $16T_0$, respectively. At the same time, the temporal correlation of input photons is clearly preserved for transport along the edge states. Similarly, Fig. 3(d-f) shows results for the $\Phi^+$ state where again the correlations are maintained at the output. We find similar results for $\Phi^-$ state. While Fig. 3 presents results for a single input frequency, the behavior remains same across the band.

To show that this preservation of temporal correlations in a 2D system is not trivial, we analyze transport through the bulk band. Unlike edge transport, the output correlation function for bulk transport varies with the input excitation frequency in the band and can also be significantly distinct from the input. Fig. 3(a-d) shows the simulated output for $\Psi^+$ input state, with $\omega - \omega_0 = (-0.52, -0.4, 0.52) J$. Interestingly, we find that the two photons can bunch at the output even though they were anti-bunched at the input. We further contrast the transport of entangled photons to that of a separable two-photon state with distinguishable photons, i.e., when there are no interference effects and the transport is essentially single-particle physics. For the separable state corresponding to state $\Psi^+$, the two-photon wavefunction is $\psi(t_1, t_2; t_e, t_l) = \phi_1 (t_1 - t_e) \phi_2 (t_2 - t_l)$, where $\phi_i (t_i)$ is the single-photon temporal wavefunction. For comparison, we symmetrize the two-photon correlation function for separable state as $\Gamma (t_1, t_2) = \frac{1}{2} \left( |\phi_1 (t_1 - t_e) \phi_2 (t_2 - t_l) |^2 + |\phi_1 (t_1 - t_l) \phi_2 (t_2 - t_e) |^2 \right)$. The simulation results are shown in Fig. 3(e-h) and we see that for separable state, bunching is much less pronounced than that for the entangled state. Similarly, Fig. 3(i-l) shows simulation results for $\Phi^+$ state where the two photons are bunched at the input, but are anti-bunched at the output. The results for a separable state corresponding to $\Phi^+$ state are shown in Fig. 3(m-p) and in this case we find that the anti-bunching is less prominent than the entangled state. This clearly demonstrates that the correlated quantum state of two photons is much more fragile than the separable state.

The temporal bunching/anti-bunching of photons seen here can be compared to spatial bunching/anti-bunching of photons which has been observed in the case of quantum walks of spatially correlated photons 25,30. Specifically, quantum walks of photons have been implemented using arrays of beam-splitters or continuously coupled waveguides, and these systems have multiple input and output ports 28,31,32. The correlated photons are coupled to different input ports and quantum walk in the system leads to spatial bunching/anti-bunching of photons at the output depending on the choice of input excitation ports and relative phase between them. In contrast, our system has a single input and single output port. But, each coupling region between the resonators is a beam-splitter and therefore, the transport of photons from input to the output by hopping this array of beam-splitters can be considered as a 2D spatial quantum walk of two photons. These spatial correlations of the two-photon quantum walk in the lattice manifest as temporal correlations at the output port.

IV. THE EFFECT OF DISORDER

Next, we study the effect of disorder on quantum transport through the edge and the bulk states. In particular, we consider a disorder in the form of on-site potential $V$ which is a result of difference in the site ring resonance frequencies. Disorder leads to localization where transmission through the system falls exponentially with the length of the system. Using transmission and delay statistics of classical light, it has been shown that the edge states are protected against disorder induced localization whereas the bulk states indeed localize 20. In contrast, here, we use non-classical, two-photon entangled states as the input and show that quantum transport through edge states is also topologically protected against localization. We consider $\Psi^+$ input state as an example where the two photons are anti-bunched at the input and show that localization manifests as temporal bunching of photons at the output.

Fig. 4(a-h) shows the simulated results at the output with input excitation in the CCW/CW edge and bulk bands, for different disorder strengths $V = (0.2, 0.4, 0.6, 0.8) J$. The correlation functions have been averaged over the corresponding bands as well as over 500 realizations of random disorder in the lattice. We can clearly see that the edge states maintain the correlation function of the input. We see some bunching only at very strong disorder strengths of $U > 0.6 J$. Also, note that this bunching is slightly more prominent for CW edge band because it travels a longer path through the lattice and is therefore more disposed to disorder. Ideally, we expect that the edge states would show no bunching at all. However, when the disorder strength is comparable to the coupling rate $J$, as is the case here, the edge bandwidth shrinks and the edge states start to lose their topological protection. On the contrary, for bulk band (Fig. 4(i-l)), even small disorder leads to significant bunching and the bunching probability increases as the disorder increases.

To quantify the effect of disorder on quantum transport, we calculate the normalized probability of bunching at the output for
FIG. 3: (a–d) Time-correlation function at the input and the output of the lattice for $\Psi^+$ state and 3 different input frequencies in the bulk band, $\omega - \omega_0 = (-0.52, -0.4, 0.52) J$. The profile is dictated largely by the input excitation frequency and the two photons can bunch at the output even when they are well separated at the input. (e–h) Correlation for separable state corresponding to the input frequencies in (a–d). For separable state, the bunching is much less than that for entangled photons. (i–p) Simulation results for $\psi^+$ state where the photons are bunched at the input and can anti-bunch at the output after propagation through bulk states. These results show that the quantum state of two entangled photons is more fragile than the separable state.

$\Psi^+$ Bell state which is anti-bunched at the input, defined as

$$ P_B = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dt_1 dt_2 \delta(t_2 - t_1 \pm \epsilon) \Gamma(t_1, t_2)}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dt_1 dt_2 \Gamma(t_1, t_2)}, $$

where two photons arriving in a time window of $\epsilon = \sigma/2$ are considered to be bunched. Fig. 3(a) shows $P_B$ as a function of disorder strength. The probability of bunching for the edge states is much less compared to that of the bulk states. Also, the probability of bunching increases with disorder strength. We further look at the similarity of the output correlation function to
FIG. 4: Correlation function at the output of a disordered system, for $\Psi^+$ input excitation in the (a-d) CCW edge band, (e-h) CW edge band and (i-l) for bulk band; for four different disorder strengths. As disorder increases, the bunching increases and is more significant for bulk band than the edge bands.

FIG. 5: (a) Calculated probability of bunching for $\Psi^+$ input state, with excitation in the edge and the bulk bands, as a function of disorder. Bunching is more prominent for bulk states and increases with disorder strength. (b) Similarity between the input and the output correlation. Even in the presence of strong disorder, the output correlation for edge states is very similar to that of the input.
the input correlation, defined as

\[
S = \frac{\left( \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dt_1 dt_2 \sqrt{\Gamma_{\text{out}}(t_1, t_2)} \Gamma_{\text{in}}(t_1, t_2) \right)^2}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dt_1 dt_2 \Gamma_{\text{out}}(t_1, t_2) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dt_1 dt_2 \Gamma_{\text{in}}(t_1, t_2)},
\]

(A8)

where \( \Gamma_{\text{in, out}} \) are the correlation functions at the input and the output. Fig. 3(b) shows the similarity \( S \) as a function of disorder strength. As expected, the correlation function at the output for edge transport resembles the input more as compared to that for bulk transport.

We have also simulated transport of \( \Psi^\pm \) states through disordered system. Similar to \( \Psi^+ \) state, we find that the edge bands preserve temporal correlations. On the other hand, for bulk transport, the two-photon anti-bunching increases with disorder, unlike \( \Psi^+ \) state where disorder leads to bunching. Note that for \( \Psi^\pm \) states, the photon are bunched at the input. Also, as we saw for \( \Psi^+ \) state, the similarity between the input and the output correlation decreases with increasing disorder strength.

V. SUMMARY

To summarize, we have studied transport of time-bin entangled photon pairs in a 2D topological photonic system. We have shown that the edge bands serve as topologically robust channels to communicate information encoded using temporal correlations of the input photons. In contrast, transport in the bulk band significantly distorts the temporal correlation function, and can result in spurious bunching/anti-bunching of photons. Using \( \Psi^+ \) state as an example, we have shown that disorder in the system leads to increased bunching for bulk states whereas for edge states, this bunching is significantly less. Similar physics could be observed in the microwave domain [33] and also in exciton-polariton systems [34]. Moreover, it is interesting to investigate the transport of entangled photons in higher dimensions [35, 36].

Appendix

In this section we show that for the linear 2D system of coupled ring resonators considered here, the two-photon temporal correlation function at the output can be calculated using the single-photon temporal wavefunction at the output. We begin with writing the general form of the time-bin entangled state of two indistinguishable photons as

\[
|\psi_{\text{in}}(t_e, t_l)\rangle = \mathcal{A} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dt_1 dt_2 \psi_{\text{in}}(t_1, t_2; t_e, t_l) \hat{a}^\dagger(t_1) \hat{a}^\dagger(t_2) |0\rangle.
\]

(A1)

Here \( t_e, l \) correspond to the early and late time bins and \( \mathcal{A} \) is a normalization factor. \( \psi_{\text{in}}(t_1, t_2; t_e, t_l) \) is the two-photon temporal wavefunction which is symmetric under exchange of photons and is given by

\[
\psi_{\text{in}}(t_1, t_2; t_e, t_l) = \frac{1}{\sqrt{2}} \left( \phi_{\text{in},1}(t_1 - t_e)\phi_{\text{in},2}(t_2 - t_l) + \phi_{\text{in},1}(t_1 - t_l)\phi_{\text{in},2}(t_2 - t_e) \right),
\]

(A2)

where \( \phi_{\text{in},i}(t_1 - t_e) \) is the single-photon temporal wavefunctions corresponding to the photon arriving in the early/late time bin, respectively. Using 2D Fourier transform, the two-photon temporal wavefunction can be rewritten in frequency domain as

\[
\hat{\psi}_{\text{in}}(t_1, t_2; t_e, t_l) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\omega_1 d\omega_2 \hat{\psi}_{\text{in}}(\omega_1, \omega_2) e^{-i\omega_1 t_1} e^{-i\omega_2 t_2},
\]

(A3)

and \( \hat{\psi}_{\text{in}}(\omega_1, \omega_2) \) is now the two-photon spectral wavefunction at the input. Using (A2),

\[
\hat{\psi}_{\text{in}}(\omega_1, \omega_2) = \hat{\phi}_{\text{in},1}(\omega_1) \hat{\phi}_{\text{in},2}(\omega_2) \left[ \exp \left( i\omega_1 t_e + i\omega_2 t_l \right) + \exp \left( i\omega_1 t_l + i\omega_2 t_e \right) \right],
\]

(A4)

where \( \hat{\phi}_{\text{in},i}(\omega) \) is the Fourier transform of single particle temporal wavefunction \( \phi_{\text{in},i}(t) \). Note that the two photons are centered around the same central frequency. For a Hamiltonian which does not have any nonlinear terms (e.g. of the form \( \hat{a}^\dagger \hat{a} \hat{a}^\dagger \hat{a} \)), there is no spectral mixing and the two-photon spectral function at the output is

\[
\hat{\psi}_{\text{out}}(\omega_1, \omega_2) = S(\omega_1) S(\omega_2) \hat{\psi}_{\text{in}}(\omega_1, \omega_2) \nonumber
\]

\[
= \hat{\phi}_{\text{out},1}(\omega_1) \hat{\phi}_{\text{out},2}(\omega_2) \left[ \exp \left( i\omega_1 t_e + i\omega_2 t_l \right) + \exp \left( i\omega_1 t_l + i\omega_2 t_e \right) \right],
\]

(A5)

where \( S(\omega) \) is the transfer function of the system for a single photon at input frequency \( \omega \), i.e., \( \hat{\phi}_{\text{out},i}(\omega) = S(\omega) \hat{\phi}_{\text{in},i}(\omega) \) and can be calculated using the input-output formalism [3]. Using the inverse Fourier transform, the two-photon temporal wavefunction
at the output can be expressed as

$$\psi_{\text{out}}(t_1, t_2; t_c, t_l) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\omega_1 d\omega_2 \tilde{\psi}_{\text{out}}(\omega_1, \omega_2) e^{-i\omega_1 t_1} e^{-i\omega_2 t_2}$$

$$= \frac{1}{\sqrt{2}} (\phi_{\text{out}, 1}(t_1 - t_c) \phi_{\text{out}, 2}(t_2 - t_l) + \phi_{\text{out}, 1}(t_1 - t_l) \phi_{\text{out}, 2}(t_2 - t_c)).$$

(A6)

Here $\phi_{\text{out}, i}(t)$ is the inverse Fourier transform of $\tilde{\phi}_{\text{out}, i}(\omega)$. Thus, we have proved that for a non-interacting Hamiltonian, the two-photon temporal wavefunction at the output can be computed from the single-photon wavefunction. Similar approach has been used earlier to study quantum walks of spatially correlated photons.

**Note:** During the preparation of this manuscript, we became aware of a similar work by Rechtsman et. al. [37]. In that work, a photonic Floquet topological system of coupled helical waveguides was studied and edge states were shown to preserve transport of path-entangled photons.

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