Confined Quasiparticle Dynamics in Long-Range Interacting Quantum Spin Chains
(Supplemental Material)

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This Supplemental Material is organized as follows. In Sec. SI, we provide detailed numerical results showing light-cone spreading of correlation functions by zooming in on Figs. 1 (b) and (c) in the main text. In Sec. SII, we provide a detailed analysis on the scaling and convergence of the potential in the main text.

SI. LIGHT-CONE SPREADING OF CORRELATION FUNCTIONS

In the main text, we have shown that the magnitude of $\langle \sigma_j^z(t)\sigma_k^z(t) \rangle_c$ is suppressed by long-range interactions. As stressed in the main text, this does not indicate the disappearance of the light-cone spreading of correlations (quantum information). In this section, we provide detailed numerics showing that the light-cone behaviour is still present by zooming in on the weak-signal regions of Figs. 1 (b) and (c) of the main text.

Figs. 1 (a)-(d) show correlation spreading after a sudden quench (for the same initial state, $|\Psi_0\rangle$, as in the main text) for several different $\alpha$. Figs. 1 (a) and (d) take the same parameters of the post-quench Hamiltonians as Figs. 1 (b) and (c) in the main text, but use an intensity scale up to two orders magnitude smaller. By zooming in on the weak-signal regions, we observe that correlations do indeed exhibit light-cone spreading, though they may spread at different maximal velocities compared to the short-range case. These results are consistent with the general theory of quench dynamics in one-dimensional systems first formulated in Refs.1–3 for short-range interacting systems, where the light-cone spreading of correlations is always present with a slope equal to twice the maximal velocity of the quasiparticles.

SII. SCALING AND CONVERGENCE ANALYSIS OF CONFINING POTENTIAL

In this section, we provide a detailed analysis on the scaling and convergence of the potential that appears in the two-kink model. We use integrals to approximate sums. While this does not give an exact value for the potential, we will see that scaling exponents given by this approximation agree well with numerics presented in the main text.

We use $V(n, L, \alpha)$ to denote the potential energy of a two-domain-wall state with length $n$ on a finite chain of length $L$. The potential can be rewritten in the following form:

$$V(n, L, \alpha) = 4 \left[ \sum_{r=1}^{L} \frac{1}{r^\alpha} + \sum_{r=2}^{L} \frac{1}{r^{2\alpha}} + \ldots + \sum_{r=n}^{L} \frac{1}{r^{n\alpha}} - 1 \right].$$

(S1)
Note that Eq. (4) in the main text can be obtained by taking the above equation to the thermodynamic limit.

We now approximate the above sums with integrals, which gives

\[ V'(n, L, \alpha) = 4 \left[ \int_1^L \frac{1}{r^\alpha} dr + \int_2^L \frac{1}{r^\alpha} dr + \ldots + \int_n^L \frac{1}{r^\alpha} dr - 1 \right] = 4 \left[ \frac{1}{\alpha - 1} \left( \sum_{\tau=1}^{n} \frac{1}{\tau^{\alpha-1}} - \frac{n}{L^{\alpha-1}} \right) - 1 \right]. \tag{S2} \]

After approximating the remaining sum, we obtain

\[ V''(n, L, \alpha) = 4 \left[ \frac{1}{\alpha - 1} \left( \int_1^n \frac{1}{r^{\alpha-1}} dr - \frac{n}{L^{\alpha-1}} \right) - 1 \right] = 4 \left[ \frac{1 - 1/n^{\alpha-2}}{(\alpha - 1)(\alpha - 2)} - \frac{n}{(\alpha - 1)L^{\alpha-1}} - 1 \right]. \tag{S3} \]

Three comments are in order: (i) The second term in the above expression tells us that, for finite \( n \), the potential is finite in the thermodynamic limit (\( L \to \infty \)) only when \( \alpha > 1 \). Therefore, the masses of these bound states are finite when \( \alpha > 1 \). This agrees with the convergence properties of the Riemann zeta function. (ii) For a finite system, the potential \( V''(n, L, \alpha) \) scales as \( c_0 - c_1/L^{\alpha-1} \). Since all the potential energies of the two-domain-wall states have such scaling, the masses given by eigenenergies of Eq. 3 in the main text should also have the same scaling. This implies that \( \beta \) (defined in the caption to Fig. 3 of the main text) is equal to \( \alpha - 1 \), which is in agreement with the numerical results presented in the inset of Fig. 3 (d). (iii) Because of the first term of the above equation, \( V(n) \) goes to infinity when \( n \) goes to infinity for \( 1 < \alpha \leq 2 \), while it is upper-bounded when \( \alpha > 2 \). This is also reflected in Fig. 3(a) of the main text. Therefore, when \( \alpha > 2 \), the two-kink model predicts that only the lower part of the energy spectrum is composed of bound states. In other words, for a high enough energy, we have a continuum of states. However, for \( \alpha \leq 2 \), all eigenstates of the two-kink model are bound quasiparticles.

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